when the atoms contain each only one electron and are independent of each other, yet is of primary importance for matter in its normal state, and without which it is impossible to explain either the magnetic properties of crystals or crystal structure. The attraction between exactly similar atoms, the directed valencies required by stereo-chemistry, and the directive forces within crystals are explicable only by a due recognition of the magnetic forcive in material media.

It is probable that the Laplace intrinsic pressure in liquids and in isotropic gels is due to electrostatic doublets of atomic dimensions. When crystallisation sets in, superimposed upon the electrostatic stresses, are the directional stresses due to the magnetic doublets which, for a given molecule, determine a characteristic crystalline symmetry, and, owing to their highly localised nature compared with the electrostatic doublets, determine the rigidity of the crystal. Such highly localised magnetic doublets suggest either that the conventional electron is moving in an orbit very small compared with atomic dimensions, or else that the electron itself is a complex unit endowed with specific magnetic as well as electrostatic properties, *i.e.*, the electron is also a magneton (*supra*, p. 271).

On the Measurement of Low Magnetic Susceptibility by an Instrument of New Type.

By Ernest Wilson, M.Inst.C.E., M.Inst.E.E.

(Communicated by Prof. J. W. Nicholson, F.R.S. Received November 5, 1920.)

It is well known in instruments of the Curie type that when the magnetising force impressed upon the substance is varied and the mechanical force is balanced against that of the torsion in a suspending fibre, the deflections follow the square law and the range through which the magnetic force can be varied is restricted. In fact, it is difficult with the Curie balance to measure the susceptibility of a given specimen outside a narrow range of magnetic force of about one to five. One object in the design of the present instrument was to be able to measure susceptibility with a given specimen through a wide range of magnetic force. Another object was to make the instrument portable in the sense that no spot of light and scale are required, and yet be able to measure susceptibility through a wide range. In all the tests so far made the instrument was placed on an ordinary table.

The fundamental idea is to replace the force due to torsion in a suspending fibre by either an electro-magnetic or electro-static system in which the mechanical force is due to two components—one proportional to the magnetic force impressed upon the specimen, and the other also proportional to the magnetic force in the case of constant susceptibility, but variable if the susceptibility varies. In the electro-magnetic case a moving coil can be employed, which is suspended in a magnetic field proportional in strength to the magnetic force acting upon the specimen. This is the method adopted in the present paper. It has the advantage that the absolute value of the susceptibility can be calculated from the known details of the instrument, and a considerable mechanical force can be produced. In the electro-static method we are concerned with the product of two voltages—one proportional to the magnetic force acting upon the specimen, and the other also proportional to this force if the susceptibility is constant, but otherwise varying with the susceptibility. This method has the advantage that the suspending fibre has only to carry a charge to the moving system, and not a definite current as in the electro-magnetic system. Also there is no question as to magnetic disturbance due to stray fields.

I. *Description of the Instrument and Apparatus.*

The electro-magnet, M (fig. 1), which produces the magnetic field in which the specimen to be tested is suspended, is capable of motion to and fro on brass rails 1, 1, of circular section. The movement is controlled by a screw, 2, which is turned by hand. The winding of the electro-magnet, M, is continued by flexible conductors to terminals, 3, fixed on the base of the instrument. The electro-magnet, M_1, which supplies the magnetic field for the moving coil, 5, is mounted on the elevated base, 4, and supported on brass pillars, 6, and its winding is connected to terminals, 7. The suspending fibre, 8, which supports the moving coil, is attached to a torsion head, 9, which also allows of an up-and-down movement for taking the moving system on and off suspension. The specimen, 10, is supported in a grip which hangs from the extremity of a beam, 11, made of aluminium wire, 0·32 cm. diameter.

The beam is rigidly fixed to the moving coil and the specimen is counterweighted by a sliding weight, 12. The horizontal distance between the centre of suspension and the centre of the specimen is 11 cm., which is therefore the effective radius. A scale, 13, fixed to the framework of the instrument, allows of any angular displacement from the true zero position being noted. As an aid to accuracy the beam is fitted with fore and back sights, 14, 15. The torsion head is carried by a horizontal brass plate, 16,

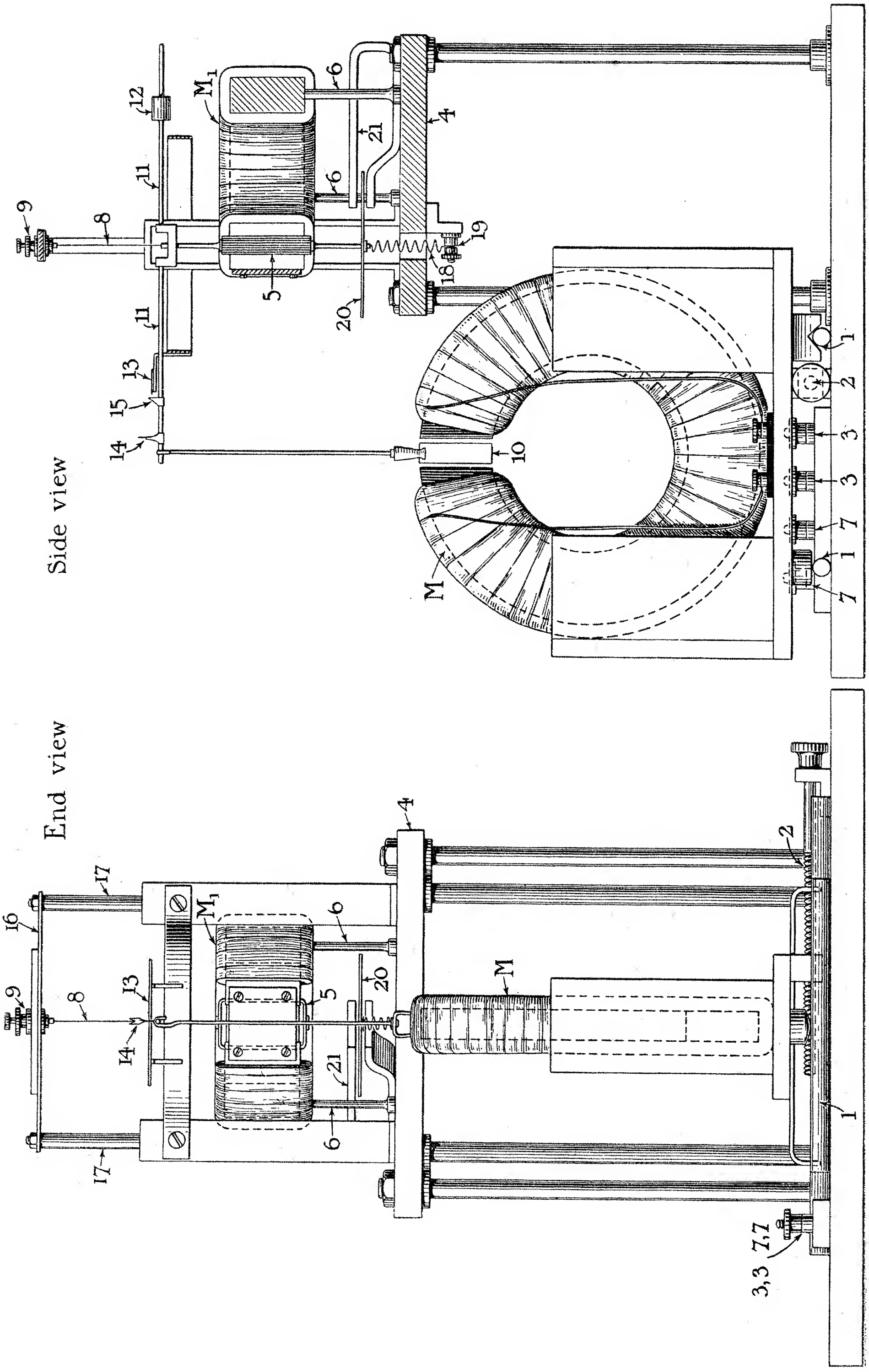
Side view
End view
M
M₁
3,3 7,7

Fig. 1.

supported at each end by pillars, 17. The current in the moving coil is carried by a conductor to the lower portion of the torsion head, 9, and passes down the suspending fibre through the coil and out by means of a spiral, 18, attached to the terminal, 19. A disc of aluminium, 20, whose diameter is 7·5 cm., is supported on an extension of the axis of the moving coil, and is free to move between the poles of a permanent damping magnet, 21. As a protection against draught, the instrument was covered by a glass shade.

A diagram of the connections is given in fig. 2, in which M and M_1 are the windings of the electro-magnets, placed in series with one another and with a shunt resistance, r_1, across which are connected an adjustable resistance, r_2, and the moving coil, r_3. In general, the current in the coil M is equal to that in M_1, as shown in fig. 2. On some occasions, however, the coil M was shunted, thus reducing the current through it. For this reason the currents in M, M_1 are represented by the symbols A, A_1 respectively. The current in the circuit is measured by the ampèremeter, A, and adjusted by means of a variable resistance, r. The current is supplied by a battery, B, and its direction through the magnet windings, M, M_1 can be reversed by the switch, S, a necessary precaution when the effects due to residual magnetism have to be removed.

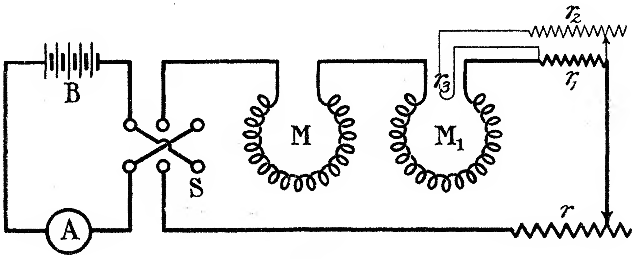

FIG. 2.

II. *The Main Magnet.*

The magnet, M (fig. 1), is the same as was used in connection with the measurement of the susceptibilities of various kinds of rock specimens,* and it is not necessary to describe it in great detail. The core is in the form of a ring of Stalloy, of rectangular section, which has an outside diameter of 18 cm. and an inside diameter of 10·5 cm. Its thickness is 1·6 cm. A radial air-space with parallel sides is cut in the ring and the width is 1·5 cm. The poles are tapered at an angle of 60° with the plane of the ring, and the opposing faces are 1 cm. wide. The winding on the ring consists of 837 turns in four layers of copper wire, 0·122 cm. diameter, insulated with two layers of cotton.

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* 'Phil. Trans.,' A, vol. 219 (Appendix, 1919), pp. 83–87 ; 'Roy. Soc. Proc.,' A, vol. 96, p. 429 (1920).

Exhaustive experiments were made to determine the distribution of the magnetic field in the air-space in a direction at right-angles to the plane of the ring, and therefrom the values of $\partial H^2/\partial x$, the gradient of the square of the magnetic force, were obtained for various currents in the magnetising coil. Further information on this subject can be obtained on reference to the original paper.

III. *The Moving Coil Magnet.*

The magnet, M_1 (fig. 1), has a core of wrought iron of rectangular section and 4 cm. deep. Its external diameter is 10 cm. and its internal diameter is 6 cm. It is wound with copper wire, 0·122 cm. diameter, doubly insulated with cotton in four layers, having a total of 301 turns. A circular air-space is cut in the ring, 2·85 cm. diameter, whose axis is parallel with the main axis of the ring. A wrought-iron cylinder, 2·35 cm. diameter and 3·25 cm. long, is supported symmetrically with the air-space by a brass plate, which bridges across the outside of the air-space. These dimensions are suitable for the insertion of a standard Weston moving coil, which was kindly given to the author by the Weston Instrument Company for the purpose of these experiments. The diameter inside the aluminium former on which the coil is wound is 2·5 cm., and the diameter over the winding is 2·66 cm. The estimated mean diameter of the winding is 2·62 cm. Effectively the turns are eighteen, and consist of two windings in parallel, each of which has eighteen turns. The resistance of the coil is 1·14 ohms.

Preliminary experiments were made with a search coil in the air-space and a ballistic galvanometer, to determine the relation between the magnetic force, H_1, and the current, A_1, in ampères in the magnetising coil. Unfortunately, for currents up to 1 ampère, the resulting curve is not a straight line, and the ring is evidently not of the best grade of annealed wrought iron. For this reason, a more elaborate treatment than is outlined in the fundamental theory has been found necessary. Had it been available, the author would have liked a ring of Stalloy, which would have given practically a straight line within the limits of the magnetising currents adopted. The relation between the magnetic force, H_1, and the ampères, A_1, is as follows, and has been taken from the mean curve:—

Ampères A_1	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0
Magnetic force H_1	40	93	153	222	290	360	427	497	566	635

IV. *Fundamental Theory.*

The relation between the magnetic force, H, in the air-space of the magnet, M, and the magnetising current, A, is very closely a straight line. Let the current in ampères be denoted by A, then $H \propto A$. H is also a function of x, the distance measured at right-angles to the plane of the magnet ring. Let this function be denoted by $f(x)$. We have then $H = qAf(x)$, where q depends neither on the ampères, A, nor the distance, x. Then, if K is the susceptibility and V is the volume of the body, the mechanical force, F, exerted by the magnetic field upon the specimen, is given by

$$F = \tfrac{1}{2}KVq^2A^2\frac{d}{dx}f^2(x).$$

Therefore for any given distance $F \propto A^2$.

If the intensity of the magnetic field for a given current in the moving coil magnet, M_1, is constant and equal to H_1, then the mechanical force in dynes, which is produced at the radius of the moving coil, is equal to $2H_1lan$; where l is the length of a single conductor, a is the current in the conductor in C.G.S. units, and n is the number of conductors. If R is the radius of the beam which supports the specimen, and r is the radius of the moving coil, the force, F, is, when a balance is obtained, equal to $2H_1lanr/R$.

The current, a, in the moving coil (fig. 2) is equal to $Ar_1/r_1+r_2+r_3$, and, since r_1 is a constant, $a \propto A/r_1+r_2+r_3$. If the force, H_1, is proportional to the magnetising current, A, the force, $F \propto Aa$, that is, it is proportional to $A^2/r_1+r_2+r_3$. But F is also proportional to KVA^2. Therefore the susceptibility, K, per unit volume is proportional to $1/r_1+r_2+r_3$, or it is equal to $C/r_1+r_2+r_3$, where C is a constant. The determination of this constant is naturally the fundamental problem for any instrument constructed after this pattern.

V. *The Torque Exerted by the Moving Coil.*

For a given current in the magnetising coil of the magnet, M_1, it was necessary to find the relation between the torque exerted by the coil when a known current was passed through it. Since the magnetic force, H_1, in the air-space is known in terms of the magnetising current, it should be possible to calculate the torque from the equation

$$T = H_1\,a\left(\frac{2\,lnr}{10}\right)k;$$

where H_1 is the magnetic force, a the current in ampères, l the length of a single conductor, n the number of conductors, r the mean radius, and k is a coefficient to correct for fringing. T is the turning moment measured in dyne-centimetres. From the particulars already given, $l = 3{\cdot}25$, $n = 18$, $r = 1{\cdot}31$, the dyne-centimetres are equal to $1{\cdot}39\ H_1 ak$.

A phosphor-bronze strip, 10 cm. long, and having a rectangular cross-section, 0·85 mm. × 0·09 mm., was used temporarily as a suspension for the moving coil. This strip had been previously tested, and is the same as used in the balance already alluded to (*loc. cit.*). It requires 1·51 dynes at a radius of 10·9 cm. to twist it through 1°. As used in the present instrument, the moving coil always occupies the position of zero torsion in the suspending fibre when the force exerted by the magnet, M, upon the specimen is a maximum; the adjustment being made by a movement of the magnet on the slide rails. It was therefore necessary to first deflect the coil by turning the torsion-head, and then to adjust the current, a, until it was brought back to zero position. The large angular deflections were read on the instrument scale, and for small angles, a spot of light from a mirror was used. The force in dynes is that exerted at 11 cm., the radius adopted in the instrument. As was to be expected, the ratio $(\text{dynes})/A_1 a$ is not constant, but experiment shows that, for a given value of the current, A_1, the ratio is also a function of a. It is therefore necessary to define the limiting values of a for given values of the ratio and magnetising current, A_1. Wider limits can be adopted when the current, A_1, is large than when it is small. By plotting the ratio $(\text{dynes})/A_1 a$, in terms of A_1, the value of the ratio for any given value of A_1 can be found. This method eliminates all errors due to the taking of dimensions and the counting of turns, and it allows of an accurate determination of the coefficient, k, in the expression for the torque above given. The values of k are not constant, but vary as follows :—

Magnetic force H_1	609	130	47	16
Values of k	1·14	1·24	1·3	1·6
Maximum values of a	0·004	0·01	0·017	0·025

VI. *The Suspending Strip.*

In the early stages of these experiments a silver strip having a resistance of 0·051 ohm per cm. was employed for suspending the moving system, and a spiral of the same strip was used to conduct the current from the moving coil. The suspending strip had a length of 5·2 cm., and the resistance of the complete moving coil system was 3·2 ohms. A length of 10 cm. of this strip requires 0·0131 dyne-cm. to produce a twist of 1°. Latterly a phosphor-bronze strip has been used for suspension 7·9 cm. long, which requires 0·00505 dyne-cm. to produce 1° of twist in a length of 10 cm. The resistance of the complete moving coil in this case is 13·73 ohms.

VII. *The Instrumental Constant.*

Exhaustive tests were made with the magnet, M (fig. 1) in order to establish the relation between the square of the magnetising current in ampères (A^2) and the force required to just remove a suspended specimen from the magnetic field against the force of torsion in a phosphor-bronze strip. It was found from the instrument itself as the mean of several determinations that the mass-susceptibility was equal to $93{\cdot}6 \times 10^{-9}\theta/A^2m$; where θ was the observed deflection and m was the mass of the specimen. The mechanical force in dynes per degree of twist at a radius of 10·9 cm. was 1·51, and the angle corresponding to unit value of θ was 0·0093 degree: so the total mechanical force was $1{\cdot}51 \times 0{\cdot}0093 = 0{\cdot}0141$ in this instance. The ratio $93{\cdot}6 \times 10^{-9}/0{\cdot}0141 = 6{\cdot}64 \times 10^{-6}$ is the constant to be used when θ is eliminated, assuming equal radii, and the expression for K_m becomes dynes $\times 6{\cdot}64 \times 10^{-6}/A^2m$. In the fundamental theory the force in dynes exerted by the moving coil is assumed to be a constant multiple of A_1a. For reasons already given this is not the case in the present instrument. This constant which we may call C, after careful determination has been plotted in terms of A_1, and from this curve the values used in the calculations have been obtained. The values of C ranged from 697×10^{-6} when the current A_1 had the value 0·0739 to 1060×10^{-6} when the current A_1 was 0·970 ampère. We have, therefore, $K_m = A_1aC6{\cdot}64 \times 10^{-6}/A^2m$ as the final expression for the mass-susceptibility.

VIII.

Manganese Sulphate.—Certain materials had to be chosen with which to test the instrument over a wide range of magnetic force, and as manganese sulphate had proved itself to be a most reliable and constant substance it was used in the first instance. The values given in the Landolt-Börnstein (1912) Tables for the mass-susceptibility (K_m) of manganese sulphate are (98, 114, 100, 85) 10^{-6} C.G.S. unit. The variation among these values is considerable, and the mean of the four is 99×10^{-6}. A glass tube containing 1·7 grm. of anhydrous manganese sulphate was tested with the magnet, M (fig. 1) using the torsion in the phosphor-bronze strip alluded to, and the value of the mass-susceptibility was found to be 0·001 very nearly.

In the first test made with the above sample of anhydrous manganese sulphate, the connections were as shown in fig. 2, and the resistance of the shunt r_1 was 2·409 ohms. As it was necessary to discover if the moving coil magnet, M_1, had any disturbing effects on the field due to the magnet, M, the currents in the moving coil and its magnet winding were reversed relatively to the current in the magnet winding of M. This change was made when the current, A, had reached the value 0·96 ampère. Assuming the susceptibility of the sulphate to be constant the values of $r_1+r_2+r_3$ should, on the theory, be also constant. For reasons already given this is not the case, and the values are now given in terms of the magnetising currents, A_1.

$A_1 = A$ ...	0·26	0·36	0·513	0·727	0·96	0·953	0·74	0·523	0·37	0·26	0·143	0·113	0·086	0·0373
$r_1+r_2+r_3$	83·6	86·6	90·6	95·3	98·6	100·6	97·6	92·6	87·6	83·6	75·6	74·1	72·6	70

In all fourteen measurements were recorded; H was varied from 25 to 642 C.G.S. units, and the corresponding change in K_m was about 11 per cent. The average value of K_m was $94·2 \times 10^{-6}$. The damping magnet was turned over to see if this had any serious effect upon the results—none was discovered.

In another series of experiments the current in the moving coil magnet winding was maintained constant during each test in which the current in the main magnet, M, was varied. Taking an average in each case the values of K_m vary as follows:—

Ampères in moving coil magnet winding A_1......	0·208	0·393	0·633	1·03
K_m	90	91·1	92·4	96×10^{-6}

The mean of these final results is $92·4 \times 10^{-6}$.

The best working range for the current in the winding of the moving coil magnet is 0·1 to 1·0 ampère. As already stated, the currents in the magnet windings of the magnets, M, M_1, are not always equal in any one test. In the case of the higher susceptibilities a shunt to the magnet, M, has been employed in order that the magnet, M_1, may be working within its best range.

Chrome Alum (2·47 grm.).—The mass-susceptibility of chrome alum was found to be 0·000014 with a force $H = 468$ when tested against the torsion of the phosphor-bronze strip. Tested in the present instrument over a range of H from 49 to 717, its value varies from 0·0000137 to 0·0000130. This substance is a valuable sub-standard when susceptibilities of small order are in question.

Iron Pyrites (5·57 grm.)—As an illustration of the measurement of low susceptibility some cubic iron pyrites was tested in powder form, and gave values of K_m varying from 0·99 to $0·979 \times 10^{-6}$ over a range of H of 168 to 740. When tested against the phosphor-bronze strip a value of $1·5 \times 10^{-6}$ was found for this variety.

Kenyte (from Cape Evans, Ross Island).—This substance is of interest in connection with the scientific work of Scott's Antarctic Expedition, 1910–1913, and the sample tested was kindly sent to the author by Dr. C. S. Wright, of the Admiralty Department of Scientific Research and Experiment. Unlike Franklinite and green serpentine which have susceptibilities of the same order of magnitude, and which like some grades of magnetite (Manchurian for example*) require a very large magnetic force to produce a maximum value of K_m, Kenyte develops an early maximum. A sample was tested as a powder against the phosphor-bronze strip and gave the following figures:—

H	26·5	48·5	72·2	103	134
K_m	253	291	318	302	300×10^{-6}

In the present experiments a sample weighing 3·08 grm. in powder form was tested at values of H ranging from 19 to 326, and the results confirm the earlier figures. The composition of this dark volcanic-looking rock is of course known, but it is interesting to note that it contains small detached portions of magnetite. Its absolute density is 2·573.

Franklinite.—The mass-susceptibility of a piece of this mineral weighing 2·69 grm. when tested in a portable instrument† was 0·00064, the value of the magnetic force, H, being 160. The present tests are in good agreement and indicate a maximum value when the force is equal to about 248 C.G.S. units.

Green Serpentine (2·50 grm.).—This rock was tested in the form of powder against the torsion of the phosphor-bronze strip and gave the following values of the susceptibility:—

H	69	95	155
K_m	135	139	169×10^{-6}

The agreement with the present experiments is satisfactory. The magnetic force, H, ranged from 19 to 270 C.G.S. units. As in the case of Franklinite the maximum susceptibility occurs at large values of the force, H.

Grey Granite.—A piece of this normal granite from Mount Sorrel, Leicestershire, has already been reported upon.‡ Its mass-susceptibility for $H = 41$ was 764×10^{-6}. When tested in the present instrument the mass-susceptibility varied from 510 to 599×10^{-6} as the force, H, was varied from 22 to 153 C.G.S. units.

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[*] 'Proc. Phys. Soc.,' vol. 31, Part V (1919).

[†] 'Proc. Phys. Soc.,' vol. 31, Part V, p. 338 (1919).

[‡] 'Phil. Trans.,' A, vol. 219 (Appendix), pp. 83–87 (1919). Also 'Roy. Soc. Proc.,' A, vol. 96, p. 429 (1920).

Copper Ferrite (2·54 grm.).—When tested over a range of H of 6·4 to 42·2, the mass-susceptibility varied from 0·0102 to 0·0169.

Ferrous Oxide (0·70 grm.).—The susceptibility (K_m) of this specimen varied from 0·0128 to 0·0211 as the force, H, was varied from 6·4 to 82·4.

In conclusion, I wish to thank Prof. E. F. Herroun for help received in the making of the instrument.

Double Refraction and Crystalline Structure of Silica Glass.

By Lord Rayleigh, F.R.S., Professor of Physics, Imperial College,
South Kensington.

(Received December 4, 1920.)

[Plates 7–9.]

§ 1. *Introduction.*

Silica glass, as is well known, may be produced by the fusion of clear crystalline quartz. In this way a clear transparent product is obtained. The present paper deals only with this kind of silica glass. The cruder variety, known as vitreosil, which is prepared from sand, is not free enough from bubbles and striæ to allow of satisfactory observation.

That silica glass may be doubly refracting was noticed in casual observations, made to test its suitability for windows, in my experiments on the scattering of light by gases. It soon became clear that this double refraction could not in all cases be due to stress, but was to be attributed to something of the nature of crystalline structure. At the same time, the double refraction is very weak indeed compared with that of crystalline quartz.

Glasses have generally been considered essentially amorphous, and, indeed, this property would usually be invoked in the definition of a glass. It may be that, in view of the present results, the definition will need to be modified, though this point is hardly ripe for discussion. In the meantime, I still use the term silica glass.

I have not met with similar effects in any of the ordinary complex glasses. When these are doubly refracting, it is always attributable to strain.

§ 2. *Method of Observation.*

Since weak double refractions are to be examined, it is necessary to have a really dark field, when looking or photographing through the crossed nicols.